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Chemical Reaction Rates and Solvent Friction

James T. Hynes[1]

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The role of the dynamic solvent friction in influencing the rates of chemical reactions in solution is described. Features considered include (a) the bias of the reaction coordinate toward a direction of lesser friction in the diffusive limit, (b) the importance of frequency-dependent friction in atom transfers, tunneling reactions and isomerizations, (c) the dynamic nonequilibrium solvation in charge transfers which leads to a polar solvent molecule reorientation time dependence for the rate, and (d) the importance of internal degrees of freedom in the location of the Kramers turnover for isomerizations.

KEY WORDS: Chemical reactions; transition state theory; Kramers theory; solvent friction; frequency-dependent friction; tunneling; Kramers turnover.

1. INTRODUCTION

The dynamic rule of the solvent in influencing the rate of an activated chemical reaction in solution has come under intense theoretical and experimental scrutiny in recent years.$^{(1-3)}$ Among the key important issues involved are (a) the validity of the most often invoked description, transition state theory (TST), in which solvent dynamics play no role, and (b) the proper way to describe the dynamic coupling of the solvent (called "friction" herein) to the reaction coordinate. There are as yet few if any definitive answers to these questions for real chemical reaction systems, but there are beginning to be fairly strong indications. In this progress, it seems fair to say that theoretical considerations have played a major role, a pleasant but perhaps atypical phenomenon.

In this paper, we will describe in fairly qualitative terms some of our own theoretical contributions to the area. We focus on the central ideas

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[1] Department of Chemistry, University of Colorado, Boulder, Colorado 80309.

and a few key results. Where possible, we indicate connections to experiments and computer simulations. (Much more exhaustive discussions with many more references can be found in Refs. 1–3.)

2. KRAMERS THEORY

Figure 1 illustrates two important classes of potential energy curves for activated solution reactions. The single barrier case is typical for atom transfer or exchange reactions of the A + BC → AB + C type. The double-well example is typical for isomerizations. Even for the latter case, we can often ignore the presence of the wells at solution densities (cf. Section 6). Then we can focus solely on the problem of the rate of passage over the reaction barrier, both for atom transfers and isomerizations.

The most venerable theory for such a passage is TST, which assumes that, in effect, collisions of the solvent molecules with the reaction system simply serve to maintain the relevant Boltzmann distributions, but do not result in any recrossing of the barrier. In this event, the rate constant is just the one-way equilibrium average flux across the barrier top ($\beta^{-1} = k_B T$)

$$k^{\text{TST}} = \langle j_+ \rangle_R = (\beta h)^{-1} \frac{Q^\ddagger}{Q_R} e^{-\beta E^\ddagger} \tag{2.1}$$

and is related to reactant (Q_R) and transition state ($Q^\ddagger$) partition functions and the activation energy in the well-known way.(4)

One important way in which the solvent can influence the rate is by a static, equilibrium modification of activation energies and other activation parameters.(5–7) This can in fact be a fairly large effect. For example, certain atom transfers are predicted(7) to be accelerated by factors of 10^2–10^3 in rare gas solvents owing to such static solvent effects. But in what follows, we will always assume that this feature has already been incorporated in

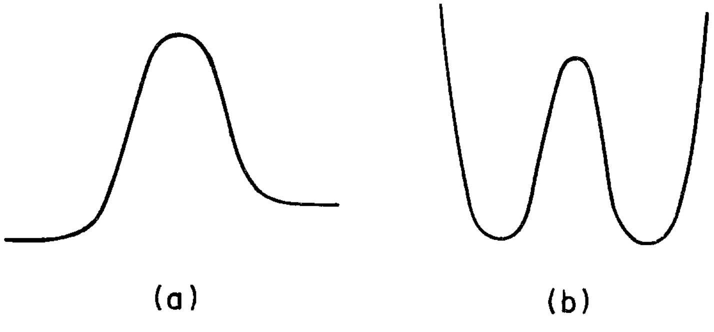

Fig. 1. Schematic potential energy curves for (a) atom transfer and (b) isomerization.

k^{TST}. Instead we will be concerned with the *dynamic* effect of the solvent in causing TST to fail in some way.

The first and still very much vital theory of the dynamic effect of the solvent on an activated barrier passage is due to Kramers in 1940.(8) Kramers first took the barrier to be parabolic with a barrier frequency ω_b [such that the potential energy $U(x) = E^{\ddagger} - \frac{1}{2}\mu\omega_b^2 x^2$, with $x^{\ddagger} = 0$ the transition state location and μ a reduced mass]. Next Kramers modeled the dynamic solvent influence by a simple one-dimensional Brownian motion or Langevin equation treatment

$$\ddot{x} = \omega_b^2 x - \zeta \dot{x} + R \tag{2.2}$$

Here ζ is the friction constant and R is a random force, related to ζ by the fluctuation–dissipation theorem.(9) (Actually Kramers used a strictly equivalent Fokker–Planck approach.) With this model, the result is the famous Kramers equation for the rate constant:

$$k/k^{\mathrm{TST}} = [1 + (\zeta/2\omega_b)^2]^{1/2} - (\zeta/2\omega_b) \tag{2.3}$$

At low friction $\zeta/2\omega_b \ll 1$, solvent collisions are negligibly effective in inducing any barrier top surface recrossing, and k approaches k^{TST}. As the friction increases, solvent buffeting of the reaction system in the barrier top neighborhood becomes more intense, barrier top recrossing is more prevalent, and k diminishes below the TST value.

In succeeding sections, we will be concerned with Eq. (2.3) as a fundamental reference point for comparison with more molecular level treatments including such features as (a) the time dependence of the actual friction experienced along the reaction coordinate, (b) the influence of any multidimensional character to the reaction, and (c) some quantum aspects.

3. THE DIFFUSIVE SMOLUCHOWSKI REGIME

When the solvent friction is sufficiently high (large ζ) and the barrier is sufficiently broad or flat (small ω_b), the rate process can be viewed as a diffusion-controlled passage over the barrier. In this "Smoluchowski" regime,(10) there is extensive solvent-induced recrossing of the barrier top and the rate constant can fall far below its TST value. According to the Kramers Eq. (2.3), k is inversely proportional to the friction constant,

$$k/k^{\mathrm{TST}} = \omega_b/\zeta \tag{3.1}$$

in this diffusive limit.

Just this behavior has in fact been observed by Keery and Fleming(11) in the isomerization of diphenyl butadiene in polar solvents. For this system, it is believed that the polar solvent stabilizes the product well in the isomerization, thereby lowering the barrier and concomitantly reducing the barrier frequency to a low value.

As we will see in Section 4, the diffusive limit is a very special case and is not easily reached for small molecule isomerization. This statement holds with even greater force for typical atom transfer reactions, to be discussed in Section 4. Nonetheless, it is probably the case that in many macromolecular systems of importance in biochemistry, there will often occur isomerizations over low broad barriers in high-friction environments. There the diffusive Smoluchowski regime or something like it will hold sway. But it is also likely that in these systems, the reaction coordinate is to some degree cooperative, i.e., several groups must act in concert for, e.g., an isomerization process to occur. The reaction is then multidimensional in an essential way, and several interesting things can happen, as we now discuss.

Perhaps the most striking manifestation of multidimensional diffusive effects is that the friction can actually *change* the reaction path.(12,13) We can illustrate the phenomenon by a simple model.(12) The model is intended to represent an intrinsic reaction event in which two inner particles are on a potential barrier, while they also are connected by potential springs to two outer particles. Each particle is supposed to be overdamped by its frictional interaction with the surroundings; but by a certain shielding effect, the friction on the inner particles is less than that experienced by the outer particles. There is thus a frictional asymmetry.

The central feature of the reaction path change can be understood as a competition between the forces derived from the potential energy on the one hand and those derived from the friction on the other. A natural initial focus is on the potential forces. This is best carried out in terms of a normal mode analysis on the model, which gives(12) the potential energy picture and normal modes shown in Fig. 2. In the reactive mode p_r, the inner particles separate in concert with the outer ones, while in the nonreactive mode p_n, the outer particles oppose the reaction.

But if we instead focus on the frictional forces, the natural description is in terms of two different frictional modes, obtained by diagonalizing the friction tensor and shown in Fig. 2. These two modes labeled f_{lo} and f_{hi} correspond, respectively, to the separation of the inner low-friction particles and the outer, higher-friction particles.

Neither of these mode pairs gives the complete picture of the reaction, which requires instead the idea of mode coupling.(12–14) For example, the reaction can proceed at a higher rate not by exclusive motion in p_r, in which high frictional forces are experienced by the outer particles, but by mixing in some p_n motion. The net effect of this mixing is to have a smaller component of the heavily damped motion of the outer particles involved in

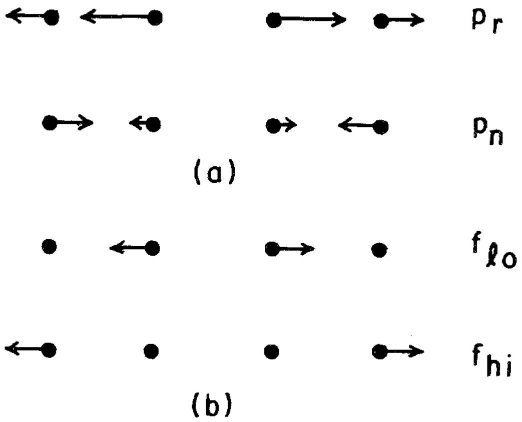

Fig. 2. (a) The reactive (p_r) and nonreactive (p_n) potential modes and (b) the frictional modes f_{lo} and f_{hi} for the four particle model described in text.

the net reactive motion. This is a bias toward lower friction. But that bias certainly cannot be continued forever, since there are restoring, i.e., nonreactive potential, forces on the p_n mode. The potential force-frictional force competition is clearly revealed here.

Figure 3 shows an actual reaction path that displays such a bias toward least friction. The result of this bias is that in extreme cases the rate

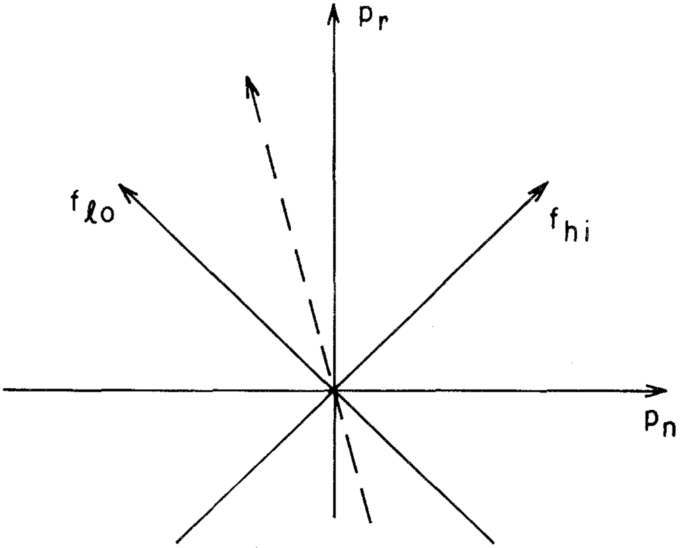

Fig. 3. Illustration of the mode coupling induced bias of the reaction coordinate (– – –) toward the direction of least friction. In the absence of coupling, reaction would be along p_r; instead it has a component along f_{lo}.

constant k can be up to an order of magnitude greater than that predicted ($\equiv k_0$) by ignoring the multidimensional aspects. These features and the mode-coupling aspects are inherent in the explicit general two-mode formula(12)

$$\begin{aligned} k/k_0 &= [2\gamma(1-\alpha)]^{-1}\{(\gamma-1)+[(1+\gamma)^2-4\gamma\alpha]^{1/2}\} \\ \alpha &= \zeta_{rn}^2/\zeta_{rr}\zeta_{nn}, \qquad \gamma=(\omega_r^2/\zeta_{rr})/(\omega_n^2/\zeta_{nn}) \end{aligned} \tag{3.2}$$

in terms of the frictional coupling parameter α in which ζ_{rn} is the frictional mode coupling, and the parameter γ measures the uncoupled diffusive motion in the normal mode force field.

More recent work(14,15) in this area has shown that when the frictional asymmetry *varies* during the barrier passage, there is a further interesting effect: the potential saddle point can be avoided altogether.

4. FREQUENCY-DEPENDENT FRICTION

As noted above, the diffusive Smoluchowski regime is generally not reached in small molecular systems at ordinary viscosities. Thus we could fall back on the Kramers result Eq. (2.3) to describe the rate constant all the way down to the low-friction values for which $k \approx k^{\mathrm{TST}}$.

But it must be recalled that the Kramers Eq. (2.3) relies on a macroscopic Brownian motion model, in which the solvent forces act instantaneously (the Markovian approximation). Not only are short-range collisions between the reaction system and the solvent molecules assumed to be instantaneous, but long-lived collective effects are also taken to act impulsively. This doubly drastic approximation is known to be poor even in nonreactive problems. It leads for example to a complete missing of the caging oscillations in velocity correlations(2) of importance for molecular translation in liquids.

The situation is even worse for the reaction problem, since there is a short time scale of critical importance. If we imagine an effective particle in an atom transfer $A+BC \to AB+C$ on the barrier top of (high-) frequency ω_b, then the strong potential barrier forces will rapidly accelerate the particle toward products. Unless the solvent forces are strong enough and rapid enough to induce recrossing in a time of order ω_b^{-1}, they are irrelevant in impeding that barrier passage. Typical barrier frequencies are fairly(20) high (10^{13}–10^{14} sec^{-1}), and the associated times are often shorter than ≈ 0.1 psec. Clearly the short time scale aspects of the solvent frictional forces need to be accounted for.

Grote and Hynes$^{(16)}$ incorporated this feature by assuming a non-Markovian generalized Langevin equation (GLE) for the reaction coordinate:

$$\ddot{x}(t) = \omega_b^2 x(t) - \int_0^t d\tau\, \zeta(t-\tau)\, \dot{x}(\tau) + R(t) \tag{4.1}$$

Here the time-dependent friction coefficient

$$\zeta(t) \propto \langle RR(t) \rangle \tag{4.2}$$

is the correlation function of the fluctuating solvent forces or torques exerted along the reaction coordinate. In this context, the Kramers theory corresponds to the unphysical δ function behavior $\zeta(t) = \zeta\delta(t)$, where the friction constant ζ gives the long time integrated effect of the solvent forces.

With the GLE description, the reaction rate constant k can be evaluated via a flux time correlation function approach developed by Northrup and Hynes.$^{(17)}$ The resulting Grote–Hynes (GH) formula for k,

$$k/k^{\mathrm{TST}} = \lambda/\omega_b = \{1 + [\hat{\zeta}(\lambda)/2\omega_b]^2\}^{1/2} - [\hat{\zeta}(\lambda)/2\omega_b] \tag{4.3}$$

has a pleasing aspect: it looks just like Kramers Eq. (2.3) but for the presence of the frequency-dependent friction

$$\hat{\zeta}(\lambda) = \int_0^\infty dt\, e^{-\lambda t} \zeta(t) \tag{4.4}$$

evaluated at the reactive frequency λ.$^{(16)}$ [The GH formula has subsequently been found in a different way by Hanggi and Mojtabai.$^{(18)}$] While λ must generally be found self-consistently, the important case where it is approximately the barrier frequency ω_b illustrates the key points. On short time scales $\approx \omega_b^{-1}$, there is often very little friction: $\hat{\zeta}(\omega_b)$ is small. This indicates first that $k \approx k^{\mathrm{TST}}$ will in fact be a very good approximation for sharp barrier reactions. (One often sees contrary statements in the literature to the effect that "TST is never a good approximation.") Second, it says that sharp barrier reactions will "track" the *short time* friction, and not be very sensitive to the longer time, hydrodynamic, or collective effects that can do so much to increase the overall long time friction constant ζ. One consequence of this is that k for a sufficiently sharp barrier reaction will not necessarily track the increasing solvent viscosity η (when in fact $\zeta \propto \eta$) as Kramers' theory would have it; rather the rate will decline at some diminished rate dependent on the details of the short time friction. Figure 4 depicts the situation.

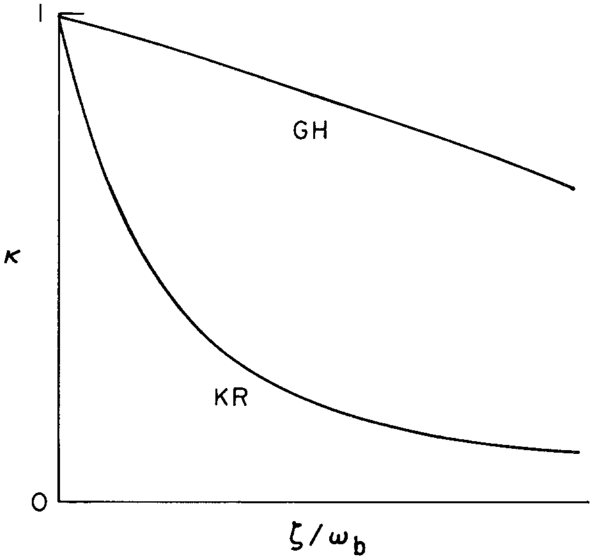

Fig. 4. Illustration of the differing behavior of κ for an atom transfer reaction according to the Kramers and Grote–Hynes descriptions.

Indeed, results similar to Fig. 4 are obtained for a model sharp barrier atom transfer rate constant calculation in which the molecular dynamics time-dependent friction of Ar(19) was modified(20) to systematically increase the collective hydrodynamic tail of $\zeta(t)$. This procedure increases the overall long time friction constant ζ, while leaving the short time collisional aspects of $\zeta(t)$ alone. The rate constant is only sensitive to the latter and not to the former as the Kramers theory would predict: k is not tracking the long time friction. (At *extremely* high friction, k will eventually begin to track it.(21))

Evidence on these predictions is gradually accumulating. For example, in a recent molecular dynamics computer study for a model 20-kcal barrier exchange reaction $Cl + Cl_2$ in liquid Ar, no solvent induced recrossing *at all* was observed.(22) An older experimental example is (possibly) provided by the activated iodine–vinyl iodide exchange reaction.(23) For this reaction, the rate is nearly the same in solvents that differ by a factor ~ 100 in viscosity.

Additional recent support has come from the isomerization experiments of the Fleming group(24) and the Barbara group.(25) The high-viscosity Kramers behavior $k \propto \eta^{-1}$ was not observed. Rather k was found to decrease less rapidly, as $k \propto \eta^{-\alpha}$ with $\alpha < 1$. In each case, a frequency-dependent hydrodynamic model friction was employed in the Grote–Hynes formula, and good agreement is obtained with the experimental results. (Other experimental results(26,27) are relevant here as well.) This is certainly very encouraging, but it must be appreciated that even now we do not know for certain what the time-dependent friction is for any reaction–solvent system. Here is a glaring lacuna that requires attention, perhaps from the point of view of molecular dynamics simulation studies.

In concluding this section, we note that there are a few interesting model cases where analytic results for κ are available.(28,29) These results may be of interest for reactions in solids and for investigating the influence of nearby polar solvent molecules in strong interaction with reacting ionic and dipolar species.(29) Perhaps the simplest soluble case is for an unstable two particle (each of mass M) oscillator on a parabolic barrier with the frequency ω_b, each harmonically coupled with frequency ω_c to an infinite chain of particles (of mass M) harmonically coupled with frequency ω_0. A particularly nice result obtains for $\omega_c = \omega_0$ and $m = M$(28):

$$\kappa = \left(\frac{2\omega_b^2}{\omega_0^2 + 2\omega_b^2} \right)^{1/2} \tag{4.5}$$

For a sharp barrier $\omega_b/\omega_0 \gg 1$, the TST result $\kappa = 1$ holds. For a broad barrier and "loose" solvent springs $\omega_b/\omega_0 \ll 1$, κ is reduced to $\sqrt{2}\omega_b^2/\omega_0^2$ by an "effective mass" phenomenon: the central reacting particle pair has to "push" the stiff surroundings out of the way. If the harmonic bath is located only on one side of the reacting pair, as for example for a reaction on a surface, then(28)

$$\kappa = \left\{ \frac{1}{2} \left[1 + \left(\frac{\omega_b^2}{\omega_0^2 + \omega_b^2} \right)^{1/2} \right] \right\}^{1/2} \tag{4.6}$$

In the stiff bath limit $\omega_b/\omega_0 \ll 1$, κ is only reduced to $(1/2)^{1/2}$. This corresponds to the feature that the effective reduced mass for the reactive pair is increased from $m/2$ to $\lim_{m_L \to \infty} [m m_L/(m + m_L)] = m$: the reaction proceeds simply by motion of that reactive particle which is located away from the bath. Related results have been worked out(28) for $A + BC$ reactions in infinite and semi-infinite chains.

5. HYDROGEN ATOM TUNNELING IN SOLUTION

The importance of the short time aspects of the friction can be revealed in another type of reaction: H atom tunneling. A good example here is the transfer of an H atom in the methyl radical–methane exchange reaction $C^*H_3 + CH_4 \rightarrow C^*H_4 + CH_3$.

The CH_3–CH_4 case is an example of a "heavy–light–heavy" system in which the light H atom moves on a time scale fast compared to the heavier flanking groups. For such systems, the dynamics are conveniently described in an appropriate skewed and scaled coordinate system (Fig. 5a). The radius ρ is approximately the heavy particle separation, while the angle θ is approximately the light H atom coordinate. In these variables, an evolving double well is apparent as the reactants approach (Fig. 5b). Simultaneously, the overall energy in ρ rises as the partners are squeezed together. The reaction can thus be considered in terms of an evolving double well, coupled to the passage in and out along the heavy particle coordinate ρ on a repulsive potential.

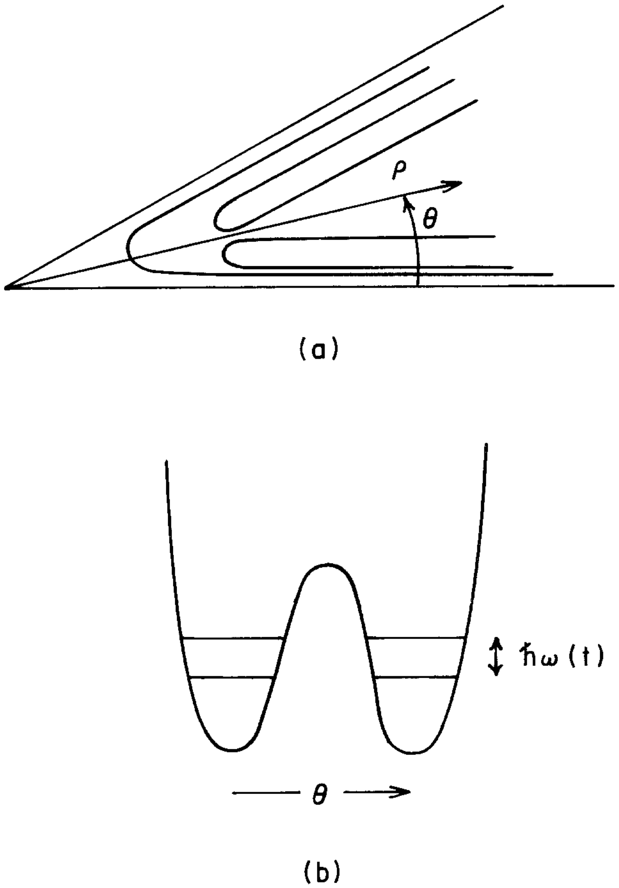

Fig. 5. (a) Skewed coordinates for a heavy-light-heavy reaction with polar coordinates indicated. (b) Evolving double well in the H atom coordinate θ as motion in ρ occurs. The tunnel splitting is indicated.

Ali and Hynes have developed a time-dependent semiclassical theory to describe the rates of such transfers in the gas phase and in solution.(30) We focus first on the gas phase. At large ρ, the double-well (DW) barrier is high, the H–CH_3 vibrations are degenerate in the two wells and there is no tunneling. As ρ decreases however, the DW barrier drops and narrows, such that tunneling is possible. The H vibrational levels split in consequence. At a *given* ρ value, a system in the reactant well will tunnel to the product well with a frequency $\omega(\rho) = \Delta E/\hbar$ (Fig. 5b) determined by the splitting $\Delta E(\rho)$: $p_{R\to P}(t) = \sin^2\{\frac{1}{2}\omega[\rho(t)]\}$. In the *dynamic* reaction problem there is a changing $\rho(t)$, and thus an evolving DW. Together with an adiabatic approximation for ρ and θ, an infinite-order perturbation theory treatment gives(30)

$$p_{R\to P}^{\text{tun}} = \sin^2 \frac{1}{2}\int_0^\infty dt\, \omega[\rho(t)] \tag{5.1}$$

for the overall tunneling probability. The reaction rate constant can be obtained by the flux correlation function formula(17)

$$k = \int_0^\infty dt \langle j_i j_0(t)\rangle_R \tag{5.2}$$

involving the incoming flux j_i into the ρ region where tunneling is possible and the subsequent outgoing flux j_0 into the ρ region where tunneling is again negligible. The average is over equilibrium initial conditions for the reactants. Evaluation of this with Eq. (5.1) gives(30)

$$k \propto \int_0^\infty dE\, p(E)\, p_{R\to P}^{\text{tun}}(E) \tag{5.3}$$

in which $p(E)$ is the equilibrium distribution for energy in the ρ coordinate.

The accuracy of Eq. (5.3) has been verified by comparison with quantum calculations.(31) The important lesson is that at 300 K, the reaction is ~90% tunneling, i.e., ~90% of the reactive trajectories tunnel below the classical barrier.

To discover how a simple inert solvent could influence this tunneling process and rate, a model was adopted in which (a) the central H atom is screened from interaction with the solvent but (b) the heavy particle classical motion along ρ in interaction with the solvent is governed by a generalized Langevin equation. Then the rate constant is given by(30)

$$k \propto \int_0^\infty dE\, p(E) \left\langle \sin^2 \frac{1}{2}\int_0^\infty dt\, \omega[\rho(t)] \right\rangle_{\text{solv}} \tag{5.4}$$

where the dynamical solvent average must be carried out. It is calculated for the $CH_3 + CH_4$ reaction in high-density Ar solvent at 300 K that k/k^{TST} is nearly the same as in the gas phase.

The essential point is that the inert solvent appears to have very little effect on the tunneling. (For example, the deuterium isotope effect should be essentially unchanged on going from gas to inert solution.) The origin of this feature is simple. The region on the steeply repulsive $E(\rho)$ curve for which tunneling can occur is small and very little time ($\sim$0.05 psec) is spent by the system there. On this time scale, the solvent has essentially no time to exert a dynamic effect. Here again we see a frequency-dependent friction effect. The short-time friction is what counts; indeed quite a different and incorrect picture results if the long-time friction is assumed to be relevant.(30)

Unfortunately, while the $CH_3 + CH_4$ reaction has been studied at higher temperatures in the gas phase,(32) no experimental results in solution are currently available. This may soon change, since the $^{13}CH_3 + {}^{12}CH_4$ reaction should be susceptible to ESR kinetic study in solution.(33)

Frequency-dependent friction is also important in more abstract treatments of quantum tunneling in a solvent or some more general bath. For example, Wolynes(34) has generalized the Kramers equation to the tunneling of a quantum particle through a parabolic barrier in contact with a bath. If a frequency-independent friction constant ζ is used, the tunneling is rapidly suppressed with increasing ζ.(34) But the application of the Wolynes formula to the parabolic barrier tunneling of a proton in polar solvents with frequency-dependent dielectric friction gives a different picture.(35) In order for the friction to be well described by a constant ζ, the barrier frequency ω_b must be fairly low. But this means that the barrier is wide and tunneling is negligible in the first place. When instead ω_b is high, the barrier is thin and tunneling is certainly important in the absence of the solvent. But at such high ω_b values, the frequency-dependent friction is small and there is little suppression of the tunneling.(35) (See the paper by Hanggi in the present issue for further discussion of tunneling.)

6. CHARGE TRANSFER REACTIONS

With the exception of Section 2, we have been concerned so far with what might be termed "weak coupling" of a neutral reaction system with a nonpolar or weakly polar solvent. We now turn to reaction rates for ionic and dipolar systems in polar solvents. Here we expect strong reaction system–solvent coupling with attendant consequences for reaction rates. Typical reaction classes in which charge redistribution or transfer occurs are S_N1 and S_N2 reactions, proton transfers, electron transfers, and *cis–trans* and other dipolar isomerizations.

The TST approach to such reactions is of course well-known.(36) It focuses on the activation free energy and its variation with reactant and transition state polarity and with the solvent polarity. Figure 6 displays a representative free energy diagram from which the solvent influence on the reaction rate would be inferred.

But this picture represents only part of the story, since it involves the assumption of *equilibrium* solvation. It is supposed that the solvent dipoles are in equilibrium at each and every point along the reaction coordinate associated with the charge distribution. While this Born–Oppenheimer-type approximation allows a discussion in terms of free energies, it ignores the fact that the solvent dipole reorientation required for this equilibration takes time and may not be complete as the charge system moves along its reaction coordinate. This state of affairs represents *nonequilibrium* solvation and leads(29,37) to a reduction of the actual rate k below its TST approximation k^{TST} to an extent related to the reorientation time of the solvent dipoles.

van der Zwan and Hynes(29,37) have examined these questions for simplified models of charge transfers and dipolar isomerizations, in which the

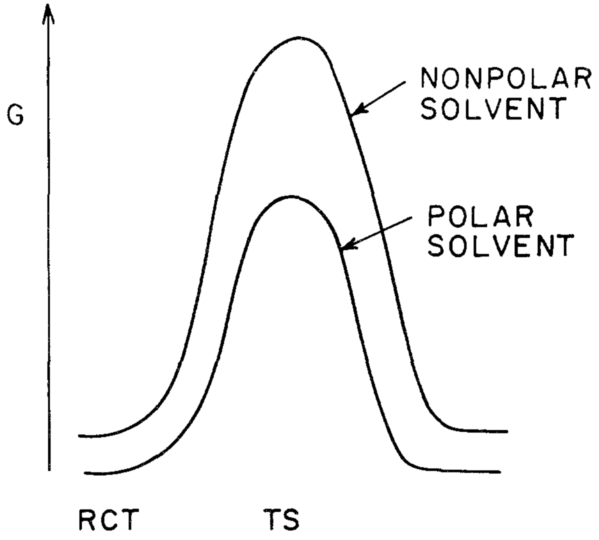

Fig. 6. Schematic free energy plot for the case where the transition state is more polar than the reactants, in nonpolar and polar solvents. The TST view is that the activation free energy is lowered in the latter case.

so-called longitudinal relaxation time τ measures the relevant solvent dipole reorientation time.(38) The theory predicts that the rate behavior will depend sensitively on whether there is strong coupling (low barrier frequency, high charge density, high solvent polarity) or weak coupling (high barrier frequency, low charge density, modest solvent polarity).

In the strong-coupling limit, the rate drops as the solvent reorientation time τ increases. In fact, in the limit of a slowly relaxing solvent, k is inversely proportional to τ. (This can be viewed as the Smoluchowski limit with the friction being of a dielectric character.(37)) Here it is the solvent motion that is rate limiting in the following sense. For large τ, the forces governing the barrier motion of, say, a charge are of two types. The chemical forces—arising from electronic molecular interactions within the reaction system—are approximately parabolic and drive the charge on to products. The solvent forces however form a "polarization cage"(37) (Fig. 7) of frequency ω_c, confining the charge to its current position. In the strong-coupling limit, the latter dominates and the charge is temporarily trapped. On any finite time scale, however, the solvent molecules reorient, the cage disappears, and the reaction eventually proceeds. The rate is thus governed by the solvent dynamics and $k \propto \tau^{-1}$:

$$k = k^{\mathrm{TST}}\, \omega_b/\omega_c^2\tau \tag{6.1}$$

Indeed this process can be described(29) in terms of motion along a solvent coordinate, which must be combined with the intrinsic charge coordinate to describe the true reaction coordinate. Equivalently stated, the solvent is a key participant in the reaction coordinate. This is a perspective totally absent in TST.

In the opposite weak-coupling limit, the rate is much less sensitive to τ. The chemical forces driving the charge down the barrier overwhelm

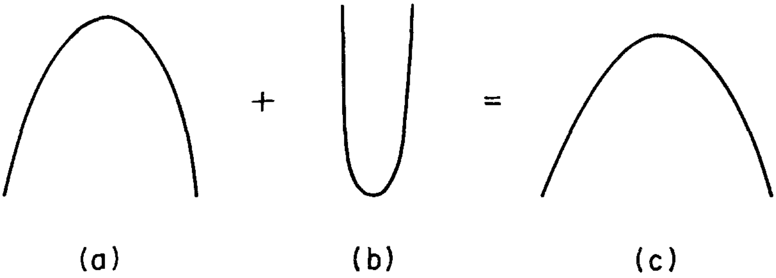

Fig. 7. Schematic illustration of the polarization cage. (a) Chemical parabolic barrier for the reaction system determined by internal electronic forces; (b) the reaction system-polar solvent caging potential; (c) the net polarization cage existing when the curvature of (b) exceeds that of (b) for a slowly relaxing solvent.

any weak polarization caging forces. Thus reaction can proceed relatively unhindered even in the presence of nearly "frozen" solvent dipoles. In fact, this is the analog of the frequency-dependent friction feature of the failure of k to track the viscosity discussed in Section 4. Here the limiting rate is in fact independent of τ(37):

$$k = k^{\mathrm{TST}}[(\omega_b^2 - \omega_c^2)/\omega_b^2]^{1/2} \tag{6.2}$$

Experimentally, there has yet to be any extensive systematic study of the dynamic role of solvent reorientation on reaction rates. But in a recent example in the Jonas group,(39) it is observed that the 1,1-difluorocyclohexane inversion, which involves a changing molecular dipole moment, is slower in more polar solvents than in less polar. While this is encouraging, all sources of friction and barrier height changes need to be sorted out. Another possible example is due to Caldin and Mateo,(40) who interpreted their proton transfer study results as a decrease in tunneling in polar solvents due to the coupling to the solvent. While this is probably intimately related to our ideas, at the moment there is no realistic dynamic theory for quantum reactions of this type in polar solvents. Finally, similar ideas emerge in electron transfer reactions,(41) and recently evidence for a $k \propto \tau^{-1}$ behavior has been found for redox reactions at electrodes by the Weaver group.(42)

Part of the theoretical problem here is our lack of real knowledge about the molecular level time-dependent friction due to Coulomb interactions. It has been proposed recently(43) that time-dependent fluorescence in polar solvents could provide some desperately needed information here. Another possible route is via molecular dynamics computer simulations; these are underway for an S_N2 reaction in water.

7. ISOMERIZATION REACTIONS

We have already described some isomerizations in the regime where all relevant dynamics occur in the barrier passage part of the problem. But beginning with Kramers,(8) the role of dynamics in the reactant and product wells in Fig. 1 has also been of concern. In our view,(2,44) this is a question of vibrational energy transfer (VET) (recall that, e.g., torsional motion is vibrational). If the friction is sufficiently low, then the VET rate of providing activated complexes on the reactant side and the VET rate of stabilizing "hot" nascent products can become rate limiting. In this regime, k falls below k^{TST} and in the extreme limit k is proportional to ζ. It is often overlooked that Kramers himself gave a simple one dimensional Brownian motion treatment of this problem in his original paper.(8) From a chemical point of view, the low friction regime is simply the bimolecular regime of a unimolecular reaction so familiar from the Lindemann and RRKM perspectives.(16,45)

Thus the overall Kramers picture for an isomerization is that starting from low-friction ζ, k will initially increase with ζ, reach a maximum somewhere near k^{TST}, and ultimately decline with ζ. In the direction of decreasing ζ, this has come to be called the "Kramers turnover." In previous sections, we have been concerned exclusively with the two latter of these three aspects; here we will focus on the first and second.

To proceed, we require an expression for k over the entire friction range. For a symmetric double well such as displayed in Fig. 1, Hynes(2) has proposed the connection formula

$$k^{-1}=k_b^{-1}+(k_d/2)^{-1} \tag{7.1}$$

Here k_b is just the barrier rate constant Eq. (4.3) which governs the barrier passage portion of the problem. Next, k_d is a VET rate constant associated with vibrational energy diffusion in the wells. The factor of 2 accounts for the requirement that both vibrational activation of the reactant and vibrational deactivation of the product is required for successful reaction. Indeed, Eq. (7.1) simply states that the reaction time is the sum of the time for activation of the reactant, the time of barrier passage, and the time for deactivation of the products. If VET is fast compared to barrier passage, then $k \rightarrow k_b$; if VET is slow compared to barrier passage, then $k \rightarrow k_d/2$ and VET is the rate-limiting step. [Asymmetric wells require a formula similar to but more complex than Eq. (7.1).(21)]

An essential feature of the VET rate constant is the role of the additional internal degrees of freedom, $s-1$ in number, coupled to the reaction coordinate. As this number increases, the VET aspect of the rate should become increasingly mute. Contrast, for example, the fate of vibrationally hot nascent product in the one-dimensional case when $s=1$ and in the multidimensional case for $s \sim 5$–10. In the former case, the solvent must perform the deactivation chore before barrier recrossing. In the latter case, presumably rapid intramolecular vibrational energy flow places energy in many degrees of freedom other than the reaction coordinate. This can subsequently be taken off by the solvent before it can find its way back to the reaction coordinate. Indeed this feature can be revealed from the k_d formula(21,46)

$$k_d^{-1}=\int_0^{E^\ddagger} dE[D(E)\,e^{-\beta E}]^{-1}\int_0^{E} dE'\,\rho(E')\,e^{-\beta E'} \tag{7.2}$$

in which $\rho(E)$ is the density of states, $E^\ddagger$ is the barrier height, and

$$D(E)=\beta^{-1}N(E)\sum_{i=1}^{s}\int_0^{\infty} dt\,\zeta_i(t)\,\pi_i(t) \tag{7.3}$$

is the VET diffusion coefficient. The latter involves the number of states up to energy E, $N(E)$, and the sum of the time-dependent friction $\zeta_i(t)$ exerted on each mode, probed at the frequencies of the mode motions determined by microcanonical mode velocity correlations $\pi_i(t)$. For $\beta E^{\ddagger} \gg 1$ and $s \gg 1$, k_d reduces very approximately to(21,46)

$$k_d \approx \frac{(\beta E^{\ddagger})^s}{s!} e^{-\beta E^{\ddagger}} Z e^{-(\omega_b \tau_c)^2} \tag{7.4}$$

where Z is the collision frequency and τ_c is the coherence time of the short-range frictional forces. In the one dimensional case, this formula emphasizes the importance of frequency-dependent friction on the rate of vibrational energy transfer between the isomerizing molecule and the solvent.(44,47) But the central feature of Eq. (7.4) to be emphasized here is the rapid growth of k_d with degrees of freedom and barrier height. The consequence of this for the rate is that(21,46) in the direction of decreasing friction, the Kramers turnover will recede to very low friction and k^{TST} will be more closely approached. In short, while many approximate one-dimensional treatments(48) predict the existence of the turnover in the liquid phases, a more realistic multidimensional treatment will predict that turnover to instead be in the gas phase. Figure 8 gives a schematic illustration.

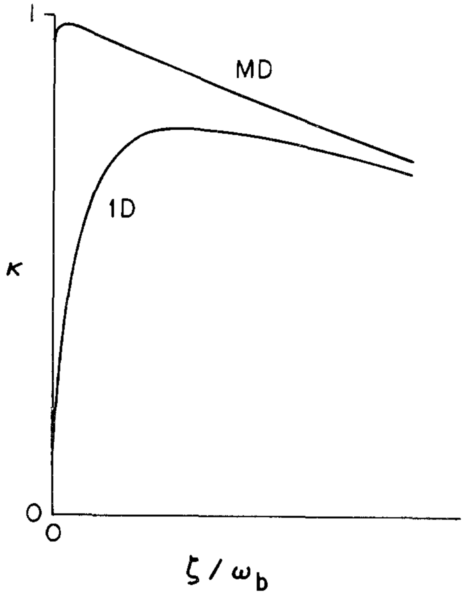

Fig. 8. Schematic illustration of the differing behavior of an isomerization rate transmission coefficient when $s=1$ [one-dimensional (1D)] and when s is about 5–10 [multidimensional (MD)].

An additional striking feature is that the turnover region, which is narrow in simple one-dimensional models, becomes considerably broader with increasing degrees of freedom.(21,46)

We have applied our theory to several cases for which experimental or computer simulation results are available, and two illustrations suffice here. For the *trans* to *gauche* butane isomerization in CCl_4, we estimate that $s \approx 3$, owing to the coupling of only low-frequency CCC bending motions to the reaction coordinate. For this case, s is low enough and the barrier height ≈ 3 kcal/mol is also low enough that the VET rise of k can occur in liquid solvent. This is in agreement with a previous molecular dynamics computer simulation result.(49)

But we believe that a liquid phase turnover will be comparatively rare—the one- or few-dimensional case is *not* the paradigm. A more typical example is stilbene isomerization, for which $E^{\ddagger}$ is about 3 kcal/mol as in butane, but with $s \approx 6$. We predict *no* turnover in the liquid phase, whereas one would be predicted from an $s = 1$ perspective. In this event, attention turns to where the turnover occurs in the gas phase. We have estimated(21) the location of the gas phase turnover for stilbene isomerization in gaseous methane at 300 K to be about 40–70 atm. This is in fact in reasonable agreement with the experimental estimates of the Fleming group.(50)

In the context of the role of the internal degrees of freedom in locating the turnover, mention must be made of the experimental results of the Jonas group(51) on solution phase cyclohexane inversion. These results have been interpreted(51) as displaying a turnover in the liquid phase, and assorted one-dimensional theories(47,52) have been applied to the reaction.

Our multidimensional calculations with $s \approx 6$(21) instead indicate that the turnover is more likely to occur in the gas phase. Further, our theory is in reasonable accord(21) with experiments on cyclohexane inversion in gas phase SF_6,(53) where it found that the rate constant apparently reaches its (gas phase) high-pressure limit at about 2–5 atm. It may be that the resolution of the questions now clearly raised about cyclohexane inversion involves a static effect: a pressure dependence of the TST activation volume(7,21) not accounted for in Ref. 51, but the issue certainly remains open.

8. CONCLUDING REMARKS

I have tried to summarize here a few of the number of ways that solvent friction and the character of that friction can influence the rate of various reaction classes in solution. Still other reaction classes, such as radical recombination,(54) could have been discussed. But it should be clear that while few definitive answers are as yet available, there are a spate of effects to look for, and questions which at least are now posed. The next few years should shed considerable light on the questions raised so far and, we hope, provide the next generations of questions to replace them.

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